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## Coulomb Drag as a Probe of Coupled Plasmon Modes in Parallel Quantum Wells

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We show theoretically that the Coulomb drag rate between two parallel quasi-two-dimensional electron gases with equal Fermi velocities is substantially enhanced by the coupled acoustic and optic plasmon modes at temperatures  $T \gtrsim 0.2T_F$  (where  $T_F$  is the Fermi temperature) for experimentally relevant parameters. The acoustic mode causes a sharp upturn in the scaled drag rate with increasing temperature at  $T \approx 0.2T_F$ . Other experimental signatures of the plasmon-dominated drag rate are a  $d^{-3}$  dependence on the well separation  $d$  and a peak as a function of relative densities at matched Fermi velocities.

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Coulomb interactions are responsible for a rich variety of phenomena in low-dimensional systems, e.g., Wigner crystallization and the fractional quantum Hall effect. However, a direct linear-response transport measurement of the effects of electron-electron ( $e$ - $e$ ) interactions in single isolated semiconductor samples is usually difficult because  $e$ - $e$  collisions conserve crystal momentum and hence do not degrade the current. It has been proposed [1,2] that when two independent electron gases with separate electrical contacts are placed in close proximity and current is driven through one, the interlayer  $e$ - $e$  interaction creates a frictional force which drags a current through the other. The magnitude of this drag force is a direct measure of the interlayer  $e$ - $e$  interactions. Drag responses between two- and three-dimensional electron gases (DEG's) [3] and between pairs of 2DEG's in  $e$ - $e$  [4,5] and electron-hole systems [6] have been seen experimentally, and these have inspired many theoretical studies [7–11]. In most experiments, no net current is allowed to flow in the layer being dragged, and a voltage arises due to charge accumulation which balances the drag-induced carrier drift. Measurement of this voltage, which is directly proportional to the interlayer drag rate, thus provides a convenient, direct, and sensitive probe of these important interlayer  $e$ - $e$  interactions.

Physically, the drag arises from an interwell transfer of momentum caused by the interlayer  $e$ - $e$  interactions. These interactions cause scattering processes in which the electrons in the driving layer (layer 1) lose momentum  $\hbar\mathbf{q}$  and energy  $\hbar\omega$ , which are gained by the electrons in the dragged layer (layer 2). The drag rate  $\tau_D^{-1} \equiv eE_2/\langle p_1 \rangle$ , where  $E_2$  is the electric field in layer 2 and  $\langle p_1 \rangle$  is the average drift momentum in layer 1, has been measured in electron-filled coupled quantum wells [4,5], for temperatures  $T \lesssim 7$  K ( $\approx 0.1T_F$ , where  $T_F$  is the Fermi temperature). At these temperatures,  $\tau_D^{-1}$  goes essentially as  $T^2$ , which agrees with the theoretical low-temperature analyses [4,7,9,10] for the interlayer momentum transfer due to Coulomb interaction. This  $T^2$  behavior is analogous to the  $e$ - $e$  Coulomb scattering rates

in a single 2DEG [12]. Deviations from the  $T^2$  behavior at around  $T = 3$  K were observed, which have been attributed to virtual phonon exchange [7,13]. However, so far experiments have not investigated effects at higher temperatures in this particular system, and the low-temperature theories for  $\tau_D^{-1}$ , when extrapolated to higher temperatures (beyond their range of validity), do not show any dramatic effects.

In this Letter, we show that for  $T \gtrsim 0.2T_F$ ,  $\tau_D^{-1}$ , in fact, has very interesting characteristics which are caused in large part by contributions of the coupled-well plasmons (collective modes). We present the following results: (1) For two identical electron gases, the scaled drag rate,  $\tau_D^{-1}/T^2$ , which is relatively constant for low temperatures [4,7,9], has a sharp upturn around  $T = 0.2T_F$  and rises to a maximum at around  $T = 0.5T_F$ . In the peak,  $\tau_D^{-1}$  is almost *an order of magnitude larger* than previously obtained results from a low-temperature approximation. In the plasmon-dominated regime, (2) the  $\tau_D^{-1}$  dependence on the (center-to-center) well separation  $d$  is approximately  $d^{-3}$ , as compared to  $d^{-4}$  in the  $T \rightarrow 0$  limit [4,7,9], and (3) at fixed  $T$ , as the relative density of the two electron gases is changed, there is a maximum in  $\tau_D^{-1}$  when the Fermi velocities of the two gases are the same.

How do the plasmons enhance the drag rate? A Coulomb scattering event is a many-body process, since it occurs in the presence of many other electrons. The scattering matrix element is therefore given by the *screened* Coulomb interaction,  $W_{12}(\mathbf{q}, \omega) \equiv V_{12}(\mathbf{q})/\epsilon(\mathbf{q}, \omega)$ , where  $V_{ij}(\mathbf{q})$  is the bare Coulomb interaction between electrons in layers  $i$  and  $j$ , and  $\epsilon(\mathbf{q}, \omega)$  is the interlayer dielectric function. The  $\epsilon(\mathbf{q}, \omega)$  can become very small (vanishing at  $T = 0$ ) for certain  $\omega(\mathbf{q})$ , corresponding to the presence of plasmons of the system. At these  $\omega(\mathbf{q})$ ,  $W_{12}(\mathbf{q}, \omega)$  becomes very large, and hence the presence of these plasmons can greatly enhance the total  $e$ - $e$  scattering in the system. In charged systems, there is generally a high frequency (optic) mode corresponding to all charges in the system oscillating in phase. In systems with two

distinct components (e.g., electron-ion or electron-hole) there can be an acoustic branch due to the charges of the two species oscillating out of phase. It has been predicted that in coupled quantum wells such an acoustic mode should exist [14]. Although coupled plasmon modes have been seen by Raman scattering in 15-layer systems [15], to the best of our knowledge, the acoustic mode has not been explicitly seen experimentally in a two-layer system. In this paper, we point out the novel possibility for observing the coupled collective modes in a transport experiment. We show that the acoustic mode causes the sharp upturn in  $\tau_D^{-1}$  and that both acoustic and optic modes contribute substantially to the drag rate. Thus, the drag rate could be used as a probe for the plasmon modes in the coupled quantum well systems.

We now describe our calculation and results. The drag rate between carriers in two parallel quantum wells, to lowest order in the interlayer interaction, is given by [6,8–10]

$$\tau_D^{-1} = \frac{\hbar^2}{8\pi^2 k_B T n_2 m_1} \int_0^\infty dq q^3 \int_0^\infty d\omega \times \frac{|W_{12}(q, \omega)|^2 \text{Im}[\chi_1(q, \omega)] \text{Im}[\chi_2(q, \omega)]}{\sinh^2(\hbar\omega/2kT)}, \quad (1)$$

where  $k_{F,i}$  and  $\chi_i(q, \omega)$  are the Fermi wave vector and two-dimensional susceptibility (including spin degeneracy) in layer  $i$ , respectively. This integral sums up the contribution of the Coulomb scattering between electrons in layers 1 and 2 over different momentum and energy transfers,  $\hbar q$  and  $\hbar\omega$ . We assume electrons only occupy the lowest energy subbands in each well, and there is negligible tunneling, which is consistent with experiments [4]. Within the random phase approximation (RPA),  $\epsilon(q, \omega)$  can be written [9,10] in terms of  $\chi_i(q, \omega)$  and  $V_{ij}(q, \omega)$ . For the form factors of  $V_{ij}$ , we assume the wells are square, with hard-wall potentials with width  $w$ .

Previous calculations on  $e$ - $e$  drag rate in coupled quantum wells [4,7–10] have concentrated on low  $T$ , and most have neglected the plasmon contribution. (Only Sivan [6] alluded to a plasmon contribution in low-density systems.) Some [4,9,10] have used the  $T = 0$  form of  $\chi_i(q, \omega)$  [16]. However, this  $\chi_i(q, \omega; T = 0)$  approximation neglects the plasmon contribution because the plasmon modes always lie outside the  $T = 0$  particle-hole continuum (as shown in the inset of Fig. 1). In this region, where  $\text{Im}[\chi(q, \omega; T = 0)] = 0$  the integrand in Eq. (1) vanishes. In contrast, for  $T \neq 0$ , the thermally activated electrons above the Fermi surface make  $\text{Im}[\chi(q, \omega)]$  finite outside the particle-hole continuum, resulting in the plasmon enhancement of  $\tau_D^{-1}$ . It is therefore essential to use the  $T \neq 0$  form of  $\text{Im}[\chi(q, \omega)]$ , and, to this end, we have developed a numerically efficient way of evaluating the full finite- $T$  RPA form of  $\chi(q, \omega)$  for arbitrary  $q$  and  $\omega$  [17], which we use to evaluate Eq. (1).

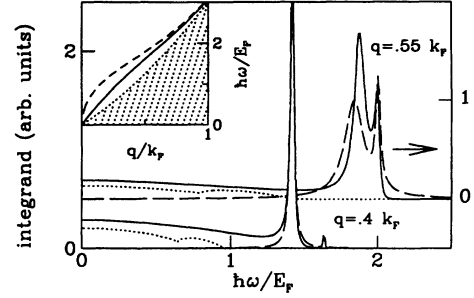


FIG. 1. Scattering rates [integrand of Eq. (1)] as a function of energy transfer  $\hbar\omega$ , at  $q = 0.4k_F$  and  $0.55k_F$ , for GaAs coupled quantum wells with equal electron densities of  $n = 1.5 \times 10^{11} \text{ cm}^{-2}$ ,  $T = 18 \text{ K}$ , well separation  $d = 3.75k_F^{-1} = 375 \text{ \AA}$ , and width  $w = 0$ . These parameters correspond to the experiment of Gramila *et al.* [4], except they had  $w = 200 \text{ \AA}$ . (Using  $w = 0$  allows a simpler evaluation of the plasmon-pole approximation contribution; results for both  $w = 0$  and  $200 \text{ \AA}$  are qualitatively very similar.) The solid (dotted) lines are for scattering rates evaluated using the finite- $T$  ( $T = 0$  approximation) form of  $\chi(q, \omega)$ . The dashed lines are the plasmon-pole approximation to the plasmon peaks. Inset: particle-hole continuum  $\text{Im}[\chi(q, \omega; T = 0)] \neq 0$  (hatched area), and acoustic and optic plasmon modes (solid and dashed lines, respectively).

The contributions to  $\tau_D^{-1}$  can roughly be divided into two categories: the single-particle part (where  $\text{Im}[\chi(q, \omega)]$  is large) and the plasmon contribution (where  $\text{Re}[\epsilon(q, \omega)] = 0$ ). For small temperatures,  $T \lesssim 0.2T_F$ , virtually the entire contribution to  $\tau_D^{-1}$  comes from the single-particle part, but as  $T$  increases above  $0.2T_F$ , the plasmon contribution becomes increasingly important. The relative contribution of these two parts is depicted in Fig. 1, which shows the integrand of Eq. (1), as a function of  $\omega$  for fixed  $q$ . The relatively flat parts of these curves which extend from  $\omega = 0$  are the single-particle contribution to the drag rate, while the two peaks are the plasmon contributions. The peak which is lower (higher) in frequency is the acoustic (optic) plasmon. A quantitative estimate of the plasmon contribution can be obtained by using a plasmon-pole approximation, which assumes that the plasmon peaks are Lorentzian. As shown in Fig. 1, for small  $q$  and  $T$ , the plasmon-pole approximation (dashed lines) is a good approximation. Even at larger  $q$  and  $T$ , when the increased damping makes the plasmon-pole approximation less accurate, it still approximates the total integrated weight well and hence is a good estimate of the total plasmon contribution to the drag rate. Note that as temperature increases above approximately  $0.5T_F$ , the plasmon peaks get extremely broad and merge into the single-particle continuum, blurring the distinction between the plasmon and single-particle contributions.

Figure 2 shows the scaled drag rate,  $T^{-2}\tau_D^{-1}(T)$ , for parameters corresponding to the experiment by Gramila *et al.* [4], calculated using both the finite- $T$   $\chi(q, \omega)$  and the  $\chi(q, \omega; T = 0)$  approximation [10]. The finite- $T$   $\chi(q, \omega)$

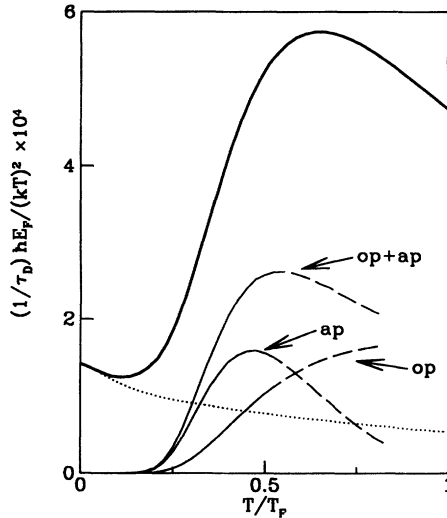


FIG. 2. Temperature dependence of the drag rate divided by  $T^2$ , for the same parameters as in Fig. 1. The full bold (dotted) curve corresponds to calculations using the finite- $T$  ( $T = 0$  approximation) form of  $\chi(q, \omega)$ . Also shown are the plasmon-pole approximation estimates for the acoustic plasmon (ap) and optic plasmon (op) contributions to the  $\tau_D^{-1}$  and the sum of the two.

curve diverges sharply at  $T \approx 0.2T_F$  from the  $\chi(T = 0)$  curve. As can be seen from the decomposition of the drag rate into components, this upturn is due in large measure to the *acoustic* plasmon, since the acoustic plasmon is lower in energy than the optic mode and hence is easier to excite thermally. As the temperatures increase, the optic plasmon also starts to contribute substantially. As  $T$  is raised further, Landau damping erodes the oscillator strength of both modes, and their contribution diminishes, causing the scaled drag rate to peak at approximately  $T = 0.5T_F$ . At extremely large temperatures, where the system is nondegenerate and the screening is negligible,  $\tau_D^{-1} \sim T^{-3/2}$  [17], but this only occurs at  $T \gg 10T_F$ .

**Dependence of  $\tau_D^{-1}$  on  $d$ .**—When there is negligible plasmon enhancement (for  $T \lesssim 0.2T_F$ ),  $\tau_D^{-1} \propto d^{-4}$  [4,7,9]. This fast falloff with  $d$  is attributable to a combination of the quasistatic (i.e.,  $\omega \ll qv_F$ ) screening of the system, which reduces the scattering rate at small  $q$ , and the interlayer Coulomb matrix element, which cuts off contributions for  $q \gg d^{-1}$ . In the regime when the plasmon enhancement dominates, however, we find that  $\tau_D^{-1}$  approximately has a  $d^{-3}$  dependence. This slower falloff in  $\tau_D^{-1}$  than in the low- $T$ , single-particle scattering dominated regime is to be expected, since the quasistatic screening which contributes to the fast  $d^{-4}$  falloff no longer applies; in fact, the plasmon enhancement can roughly be thought of as an antiscreening of the Coulomb interaction. (It is difficult, however, to extract an exact analytic expression for the  $d$  dependence at plasmon-dominated temperatures.) In Fig. 3(a), we show  $d^4 T^{-2} \tau_D^{-1}(T)$  for different  $d$ . At low  $T$ , the curves con-

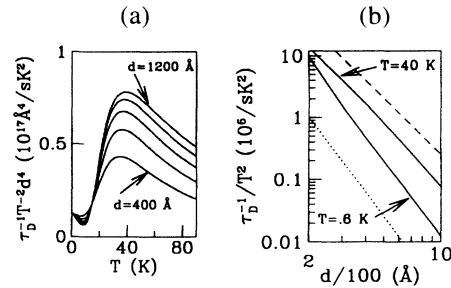


FIG. 3. (a) Scaled drag rates  $\tau_D^{-1} d^4 / T^2$  as a function of temperature, for different well separation  $d$ . Parameters used are as in Fig. 1, but now with  $w = 200$  Å. The lowest curve is for  $d = 400$  Å, the highest for  $d = 1200$  Å, and successive curves in between differ by 200 Å. (b) Dependence of  $\tau_D^{-1} / T^2$  on  $d$  for  $T = 0.6$  and 40 K. The dashed and dotted lines are reference  $d^{-3}$  and  $d^{-4}$  curves, respectively. The drag rate at  $T = 40$  K, in the plasmon-dominated regime, has approximately a  $d^{-3}$  dependence for  $d \lesssim 700$  Å, compared to the  $d^{-4}$  dependence at  $T = 0.6$  K in the single-particle dominated regime.

verge at the same point, showing the  $d^{-4}$  scaling. However, the relative size of the peak grows larger with increasing  $d$ , showing a slower falloff than  $d^{-4}$ . Figure 3(b) explicitly shows the  $d^{-3}$  falloff in  $\tau_D^{-1}$  in the plasmon-dominated regime.

**Dependence of  $\tau_D^{-1}$  on relative densities.**—The  $\tau_D^{-1}$  shows a peak when the two Fermi velocities  $v_{F,i}$  are equal (as compared to  $k_{F,1} = k_{F,2}$  for phonon-mediated drag [5,7]). Figure 4 shows peaks in  $\tau_D^{-1}$  at matched densities for identical wells and at  $n_2/n_1 = (m_2^*/m_1^*)^2$  when the effective masses  $m_i^*$  in the two wells are different. The reason for this is as follows: Since  $\tau_D^{-1}$  depends on the product  $\text{Im}[\chi_1] \text{Im}[\chi_2]$  [see Eq. (1)], the plasmon modes must be close in energy to the particle-hole continua of *both* layers 1 and 2 if the plasmons are to enhance the scattering of the thermally excited carriers. The plasmon modes always lie above the  $T = 0$  particle-hole continuum of the electron gas with the larger Fermi velocity (otherwise, they would be heavily Landau damped). Thus if  $v_{F,1} \gg v_{F,2}$ , then the phase velocity of the plasmons is much higher than  $v_{F,2}$ , making it difficult for the scattering of particles in layer 2 to be enhanced by the plasmons. The optimum situation for plasmon enhancement clearly occurs when  $v_{F,1} = v_{F,2}$ . As shown in Fig. 4, the peak at  $v_{F,1} = v_{F,2}$  is most pronounced at large  $d$ , since the relative plasmon enhancement increases with  $d$ .

We now briefly discuss the approximations and assumptions we have used. First, there is evidence that the RPA breaks down when the  $r_s$  parameter is greater than 5 for large  $q$  [18]. In the experiment in Ref. [4] on which the calculations in paper are mainly based,  $r_s \approx 1.4$ , and most of the contribution to the drag rate comes from  $q \lesssim 0.5k_F$ , so RPA should still be valid here. We have also ignored virtual phonon exchange [5,7,13]. This

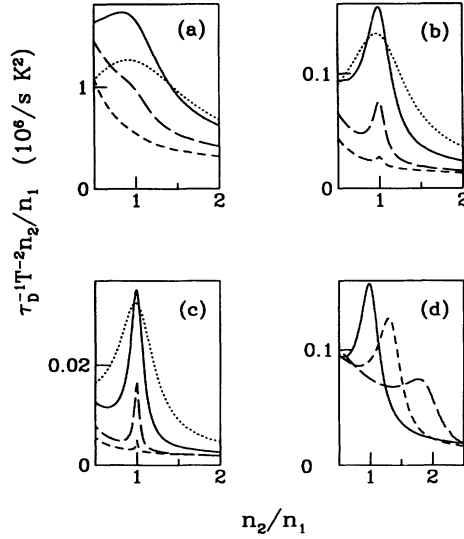


FIG. 4. Scaled drag rates  $\tau_D^{-1} T^{-2} n_2/n_1$  as a function of the ratio of well carrier densities,  $n_2/n_1$ , where  $n_1 = 1.5 \times 10^{11} \text{ cm}^{-2}$ , for (a)  $k_{F,1}d = 4$ , (b)  $k_{F,1}d = 8$ , (c)  $k_{F,1}d = 12$ . (The factor  $n_2/n_1$  accentuates the plasmon peak by partly suppressing the rise in  $\tau_D^{-1}$  as  $n_2 \rightarrow 0$ .) The short dashed, long dashed, dotted, and solid lines correspond to temperatures  $T = 12, 18, 30$ , and  $60$  K, respectively. (d) Scaled drag rate at  $k_{F,1}d = 8$  at  $T = 30$  K for various ratios of effective masses in the different wells. The solid, short dashed, and long dashed lines are for  $m_2^*/m_1^* = 1, 1.2$ , and  $1.4$ , respectively. The maximum plasmon enhancement occurs when  $v_{F,1} = v_{F,2}$ , i.e.,  $n_2/n_1 = (m_2^*/m_1^*)^2$ .

virtual phonon mechanism gives a significant contribution at around  $T = 3$  K, but its relative contribution to  $\tau_D^{-1}/T^2$  decreases with increasing  $T$ . Thus, since the maximum of the plasmon enhancement occurs at approximately 30 K, the virtual phonon contribution at this temperature should be negligible. However, since the virtual phonon exchange is weakly dependent on  $d$ , the virtual phonon exchange tends to dominate for large  $d$ , but we estimate that the plasmon enhancement should not be masked for  $k_F d \lesssim 10$ .

Before concluding, we address why a plasmon enhancement is not clearly seen in existing high-temperature drag data of Refs. [3] and [6]. In the electron-hole system [6], since the hole is approximately 6.7 times heavier than the electrons, the enhancement would occur at the electron to hole density ratio of  $1/45$ , which is well outside the range of their data. In the two- to three-dimensional system [3], the interpretation of the data is complicated by various extraneous effects such as Joule heating and thermoelectric effects, which mask the Coulomb drag effect.

In conclusion, we have calculated the drag rate for coupled quantum wells, taking into account the effect of plasmons through proper treatment of the dynamical screening of the coupled electron gases. The plasmons of the system greatly enhance the drag rate for tempera-

tures above  $0.2 T_F$ . We have elucidated the unique experimental signatures of this plasmon enhancement in the scaled drag rate,  $\tau_D^{-1}/T^2$ , which include (1) a sharp upturn at  $T \approx 0.2 T_F$  and a maximum at  $T \approx 0.5 T_F$  in  $\tau_D^{-1}/T^2$ , (2) a  $d^{-3}$  dependence in  $\tau_D^{-1}$ , and (3) a peak at matched Fermi velocities of the two wells.

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